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Common Origin of $(-)^{\mathrm{L}}$, $(-)^{\mathrm{3B}}$, and Strong CP Conservation

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Abstract

The multiplicative conservation of both lepton and baryon numbers, i.e. $(-)^L$ and $(-)^{3B}$, is connected to an axionic solution of the strong CP problem in a supersymmetric, unifiable model of quark and lepton interactions. New particles are predicted at the TeV scale, with verifiable consequences at the Large Hadron Collider.

Experimentally, there is no evidence against the conservation of additive lepton (L) and baryon (B) numbers. Nevertheless, the prevailing theoretical thinking is that neutrino masses are Majorana and only $(-)^L$ is conserved. This implies the occurrence of neutrinoless double beta decay [1] which is being pursued actively but yet to be confirmed. Recently it has been pointed out [2] that the parallel situation of $(-)^{3B}$ conservation is also possible, with the consequence of an absolutely stable proton but allowing deuteron decay and neutron-antineutron oscillations. In the following these multiplicative conservation laws are connected to an axionic solution of the strong CP problem in a supersymmetric, unifiable model of quark and lepton interactions. Heavy quarks of charge $\mp 1/3$ with $B = \mp 2/3$ at the TeV scale are predicted.

The idea of $(-)^L$ conservation is well-known. The neutrino ν ($L = 1$) is paired with a singlet neutral fermion N^c ($L = -1$) through the standard Higgs doublet (ϕ^+, ϕ^0). In the presence of electroweak $SU(2)_L \times U(1)_Y$ symmetry breaking, $\langle \phi^0 \rangle = v$ implies a Dirac mass m_D linking ν with N^c. However, N^c is a gauge singlet and as such, it is allowed a large Majorana mass m_N, thereby breaking L to $(-)^L$ and resulting in a small seesaw Majorana mass for ν, i.e. $m_\nu = m_D^2/m_N$.

Similarly, the idea of $(-)^{3B}$ conservation requires a singlet neutral fermion Σ, carrying $B = 1$ in the effective interaction

$$u_i^c d_j^c d_k^c \Sigma.$$

To implement this in a renormalizable theory, the simplest way is to introduce singlet scalar quark fields $\tilde{h}, \tilde{h}^c$ with charges $\mp 1/3$ and $B = \mp 2/3$, so that the interactions $u^c d^c \tilde{h}^c$ and $\tilde{h} d^c \Sigma$ are allowed. Alternatively, $\tilde{h}, \tilde{h}^c$ may be assigned charges $\pm 2/3$, in which case $d^c d^c \tilde{h}^c$ and $\tilde{h} u^c \Sigma$ are allowed. The large Majorana mass m_Σ breaks B to $(-)^{3B}$ under which the usual quarks are odd and the exotic scalar quarks $\tilde{h}, \tilde{h}^c$ are even.

The decay of the lightest N^c in the early Universe generates a lepton asymmetry, whereas

the decay of the lightest Σ generates a baryon asymmetry. Both are converted to a $B-L$ asymmetry from the intervention of electroweak sphalerons [3]. Below the scale of m_N, all particle interactions conserve additive L except for rare processes involving the effective exchange of N^c such as neutrinoless double beta decay. Below the scale of m_Σ, all particle interactions conserve additive B except for rare processes involving the effective exchange of Σ such as deuteron decay and neutron-antineutron oscillations. The two scales m_N and m_Σ are not *a priori* related, but in the context of an axionic solution of the strong CP problem, both will come from the vacuum expectation value of a singlet field, the dynamical phase of which contains the axion, as shown below. A unifying picture is thus possible for the common origin of $(-)^L$, $(-)^{3B}$, and strong CP conservation.

The strong CP problem is the appearance of the instanton-induced term [4, 5]

$$\mathcal{L}_\theta = \theta_{QCD} \frac{g_s^2}{64\pi^2} \epsilon_{\mu\nu\alpha\beta} G_a^{\mu\nu} G_a^{\alpha\beta} \tag{1}$$

in the effective Lagrangian of quantum chromodynamics (QCD), where g_s is the strong coupling constant, and

$$G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu + g_s f_{abc} G_b^\mu G_c^\nu \tag{2}$$

is the gluonic field strength. This term is odd under CP and if θ_{QCD} is of order unity, the neutron electric dipole moment would be 10^{10} times its present experimental upper limit $(0.63 \times 10^{-25} e$ cm) [6]. This undesirable situation is most elegantly resolved by invoking a dynamical mechanism [7] to relax the above θ_{QCD} parameter (including all contributions from colored fermions) to zero. However, this requires an anomalous global $U(1)_{PQ}$ symmetry which is broken at the scale f_a and results necessarily [8, 9] in a very light pseudoscalar particle called the axion, which has not yet been observed [10].

To reconcile the nonobservation of an axion in present experiments and the constraint 10^9 GeV $< f_a < 10^{12}$ GeV from astrophysics and cosmology [11], three types of "invisible"

axions have been discussed. (I) The DFSZ solution [12, 13] introduces a heavy singlet scalar field as the source of the axion but its mixing with the doublet scalar fields (which couple to the usual quarks) is very much suppressed. (II) The KSVZ solution [14, 15] also has a heavy singlet scalar field but it couples only to new heavy colored fermions. (III) The gluino solution [16] identifies the $U(1)_R$ of superfield transformations with $U(1)_{PQ}$ so that the axion is a dynamical phase attached to the gluino (which contributes to θ_{QCD} because it is a colored fermion) as well as all other superparticles.

In a supersymmetric extension of the Standard Model, it is also important that the breaking of $U(1)_{PQ}$ at the large scale f_a does not break supersymmetry as well. This may be accomplished using three singlet superfields in various ways, for the gluino solution [17, 18, 19] and for the DFSZ solution [20]. In Table 1, the $(-)^L$, $(-)^{3B}$, and PQ charges of the superfields of this construction are listed.

Table 1: Particle content of proposed model.

Superfield	$SU(3)_C \times SU(2)_L \times U(1)_Y$	$(-)^L$	$(-)^{3B}$	$U(1)_{PQ}$
$Q \equiv (u, d)$	$(3, 2, 1/6)$	+	−	1/2
u^c	$(3^*, 1, -2/3)$	+	−	1/2
d^c	$(3^*, 1, 1/3)$	+	−	1/2
Σ	$(1, 1, 0)$	+	−	1/2
$L \equiv (\nu, e)$	$(1, 2, -1/2)$	−	+	1/2
e^c	$(1, 1, 1)$	−	+	1/2
N^c	$(1, 1, 0)$	−	+	1/2
$\Phi_1 \equiv (\phi_1^0, \phi_1^-)$	$(1, 2, -1/2)$	+	+	−1
$\Phi_2 \equiv (\phi_2^+, \phi_2^0)$	$(1, 2, 1/2)$	+	+	−1
h	$(3, 1, -1/3)$	+	+	−1
h^c	$(3^*, 1, 1/3)$	+	+	−1
S_2	$(1, 1, 0)$	+	+	2
S_1	$(1, 1, 0)$	+	+	−1
S_0	$(1, 1, 0)$	+	+	−2

The most general superpotential with this particle content is then given by

$$\begin{aligned} W &= m_0 S_0 S_2 + \lambda_1 S_1 S_1 S_2 + \lambda_2 S_1 N^c N^c + \lambda_3 S_1 \Sigma\Sigma \\ &+ f_1 S_2 \Phi_1 \Phi_2 + f_2 S_2 h h^c + f_3 QQh + f_4 u^c d^c h^c + f_5 h d^c \Sigma \\ &+ f_d \Phi_1 Q d^c + f_u \Phi_2 Q u^c + f_e \Phi_1 L e^c + f_N \Phi_2 L N^c. \end{aligned} \tag{3}$$

Note that the only allowed mass term is m_0 which is thus expected to be large. With W of Eq. (3), it has been shown [20] that it is possible to break $U(1)_{PQ}$ spontaneously at the scale m_0 without breaking the supersymmetry. The soft breaking of supersymmetry will then introduce another (much smaller) scale M_{SUSY}, with the result $u_1 = \langle S_1 \rangle$ and $u_0 = \langle S_0 \rangle$ are of order m_0, whereas $u_2 = \langle S_2 \rangle$ is of order M_{SUSY}. This means that the so-called μ problem in the Minimal Supersymmetric Standard Model (MSSM) is solved because $\mu = f_1 u_2$. Similarly, the exotic h quark has the mass $f_2 u_2$ and should be observable at the Large Hadron Collider (LHC). As for the masses of N^c and Σ, they are given by $2\lambda_2 u_1$ and $2\lambda_3 u_1$ respectively, with the axion contained in the dynamical phase of S_1. Hence a common origin emerges for the conservation of $(-)^L$, $(-)^{3B}$, and strong CP.

To see how supersymmetry remains unbroken at the axion scale, consider the scalar potential of $S_{2,1,0}$, i.e.

$$V = m_0^2 |S_2|^2 + 4\lambda_1^2 |S_1|^2 |S_2|^2 + |m_0 S_0 + \lambda_1 S_1^2|^2. \tag{4}$$

There are two supersymmetric minima: the trivial one with $u_0 = u_1 = u_2 = 0$, and the much more interesting one with

$$u_2 = 0, \quad m_0 u_0 + \lambda_1 u_1^2 = 0. \tag{5}$$

The latter breaks $U(1)_{PQ}$ spontaneously and shifting the superfields by $u_{2,1,0}$, the superpotential of $S_{2,1,0}$ becomes

$$W' = \frac{m_0}{u_1}(u_1 S_0 - 2u_0 S_1) S_2 + \lambda_1 S_1 S_1 S_2, \tag{6}$$

showing clearly that the linear combination

$$\chi = \frac{u_1 S_1 + 2u_0 S_0}{\sqrt{u_1^2 + 4u_0^2}} \tag{7}$$

is a massless superfield.

At this point, the individual values of u_1 and u_0 are not determined. This is because the vacuum is invariant not only under a phase rotation but also under a scale transformation as a result of the unbroken supersymmetry [21], i.e. a flat direction. As such, it is unstable and the soft breaking of supersymmetry at M_{SUSY} will determine u_1 and u_0 separately, and u_2 will become nonzero. Specifically, the supersymmetry of this theory is broken by all possible holomorphic soft terms which are invariant under $U(1)_{PQ}$. As a result [20],

$$u_2 \sim M_{SUSY}, \quad m_0 u_0 + \lambda_1 u_1^2 \sim M_{SUSY}^2, \tag{8}$$

with u_0 and u_1 individually of order m_0.

As the electroweak $SU(2)_L \times U(1)_Y$ gauge symmetry is broken by the vacuum expectation values $v_{1,2}$ of $\phi^0_{1,2}$, the observed doublet neutrinos acquire naturally small Majorana masses given by $m_\nu = f_N^2 v_2^2/(2\lambda_2 u_1)$ by way of the usual seesaw mechanism. Since $\phi^0_{1,2}$ have PQ charges as well, the axion field is now given by

$$a = V^{-1}\left[u_1\theta_1 + 2u_0\theta_0 - 2u_2\theta_2 + \frac{2v_1v_2}{v_1^2+v_2^2}(v_1\varphi_2 + v_2\varphi_1)\right], \tag{9}$$

where $V = [u_1^2 + 4u_0^2 + 4u_2^2 + 4v_1^2v_2^2/(v_1^2+v_2^2)]^{1/2}$, and θ_i, φ_i are the various properly normalized angular fields of the corresponding complex scalars. The axionic coupling to quarks is thus

$$\begin{aligned}
&(\partial_\mu a)\frac{1}{2V}\left(\frac{2v_1v_2}{v_1^2+v_2^2}\right)\left[\frac{v_1}{v_2}\bar{u}\gamma^\mu\gamma_5 u + \frac{v_2}{v_1}\bar{d}\gamma^\mu\gamma_5 d\right] \\
&= \frac{1}{V}(\partial_\mu a)[\sin^2\beta\ \bar{u}\gamma^\mu\gamma_5 u + \cos^2\beta\ \bar{d}\gamma^\mu\gamma_5 d],
\end{aligned} \tag{10}$$

where $\tan\beta = v_1/v_2$, as in the DFSZ model.

The scale m_2 determines the axion scale as well as m_N and m_Σ. The decay of the lightest N^c generates a lepton asymmetry whereas the decay of the lightest Σ generates a baryon asymmetry [2]. Below their common mass scale, both L and B are conserved additively, hence each asymmetry will be converted to a $B-L$ asymmetry through the interaction of the electroweak sphalerons [3]. Whereas thermal equilibrium of leptons is affected by their known Yukawa couplings (flavor dependence), baryogenesis through Σ decay may be more efficient because the QQh and $u^c d^c h^c$ couplings may not necessarily be small. If kinematically allowed, $\tilde{h}$ and $\tilde{h}^c$ will be produced in abundance at the LHC. Their decay into 2 quark jets may have a chance of being observed.

The appearance of the exotic h and h^c superfields at the M_{SUSY} scale, presumably of order TeV, would spoil the gauge-coupling unification of the MSSM at around 10^{16} GeV. To remedy this situation, a very simple solution is to use four Higgs doublets instead of two. This is easily accomplished by assigning them separately to the quark and lepton sectors: two to d^c and u^c, and two to e^c and N^c. Now the two extra Higgs doublets combine with h and h^c to form complete multiplets of $\underline{5}$ and $\underline{5}^*$ under $SU(5)$, thereby preserving the gauge-coupling unification of the MSSM. It also means that the phenomenology of the supersymmetric Higgs sector becomes much richer.

Since $(-)^L$ and $(-)^{3B}$ remain conserved, so is the usual R parity of the MSSM. The neutralino mass matrix is now 9×9 instead of 4×4 because of the two additional neutral higgsinos as well as $S_{2,1,0}$. As shown in Eq. (6), two of these fields combine to form a heavy Dirac fermion at the m_0 mass scale, leaving seven at the TeV scale. One linear combination is then the axino, but its mixing with the other neutralinos is small, i.e. of order $v_{1,2}/V$. The lightest among these seven particles is a candidate for the dark matter of the Universe, in addition to the axion.

In conclusion, it has been pointed out in this note that the origin of the multiplicative

conservation of lepton and baryon numbers may be the same as that of strong CP conservation. They are all related to the complex singlet superfield S_1 whose vacuum expectation value determines the axion scale as well as the mass scale at which L breaks to $(-)^L$ and B breaks to $(-)^{3B}$. There are then two sources for the baryon asymmetry of the Universe, thus relieving some of the tension inherent in the usual leptogenesis scenario [3]. Only one other mass scale appears in this scenario, i.e. M_{SUSY}, which explains why the electroweak breaking scale cannot be too far from it and bolsters the expectation that supersymmetry will be discovered at the LHC. The implementation of $(-)^{3B}$ conservation predicts new exotic particles h, h^c at the TeV scale, presumably together with two more Higgs doublets if gauge-coupling unification of the MSSM is to be maintained. There are seven neutralinos (including the axino) at the TeV scale, the lightest of which is a dark-matter candidate, in addition to the axion.

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